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A cavity-wall element for Statistical Energy Analysis of sound transmission through double-wall structures

Svante Finnveden
ACAD International AB, Sveavägen 151, 113 46 Stockholm, Sweden. svante.finnveden@acad.se

The wave motion within a cavity between two flexible walls is first investigated numerically. In a frequency region between the double wall resonance and a frequency at which the cavity depth is approximately half an acoustic wavelength, there is only one kind of wave in the fluid and two flexural waves in the walls which induces some near field motion in the fluid. The fluid wave includes some mass impeded wall motion and is supersonic, so it radiates well into a connecting room. Based on these findings, a new SEA formulation is derived in which each element approximately describe one kind of coupled wall-cavity-wall wave motion. This formulation obsoletes the non-resonant transmission path commonly used in SEA of sound transmission and compared to classical formulations it improves results at frequencies around and a bit above the double wall resonance. The new formulation is compared to three sets of measurements found in the literature showing fair agreements.

1 Introduction

Double walls are frequently used in buildings, vehicles and aircraft and continue to be of interest for research [1]-[6]. The classical SEA double wall formulation describes the walls and the cavity as three separate elements and includes a direct, non-resonant, path from a room to the cavity [7]-[8]. Here, a new SEA formulation is derived based on the wave motion of the coupled cavity-wall structure. The double wall formulation considers a partition of size $S = L_x \times L_y$ consisting of two plates separated by a distance $L_z = d$. For both plates, the coincidence frequencies, at which the flexural wavelength of plate i equals the acoustic wavelength, occurs at a rather high frequency. In a mid-frequency regime, the plates' impedances to acoustic forcing are therefore predominantly of mass character. The double wall resonance, f_{dw}, is mainly determined by the wall's masses and the air stiffness of the cavity, all per unit area; it is approximately given by

$$f_{dw} = \frac{c}{2\pi d}\sqrt{\frac{1}{\mu_1}+\frac{1}{\mu_2}} \tag{1}$$

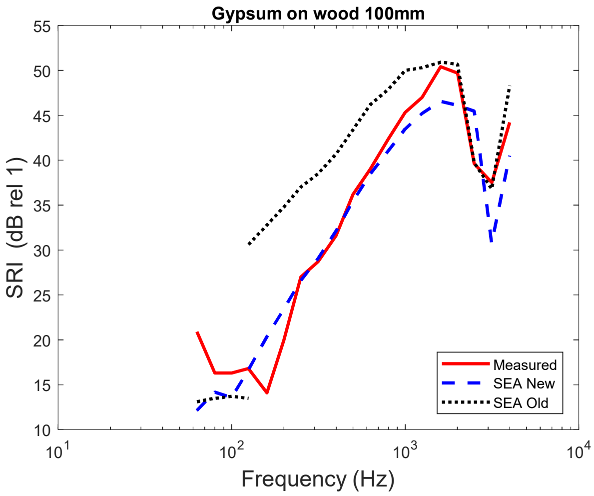
where c is the sound speed and $\mu_i = \rho_i t_i / \rho_0 d$ and ρ_i and t_i are the density and wall thickness of plate i and ρ_0 is the air density. At frequencies lower than f_{dw}, the plates are assumed to move in phase and the transmission loss is approximately given by the mass law employing the total mass of the partition.

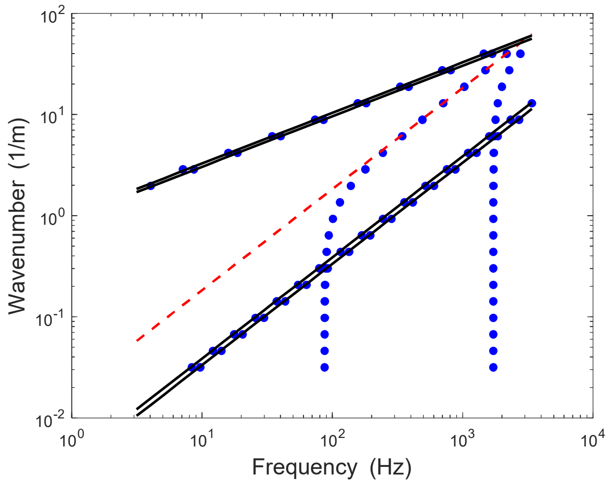
At frequencies above f_{dw}, Price and Crocker [7] define five SEA elements, each describing the response in one of the sub structures: 1) sending room; 2) plate 1; 3) cavity; 4) plate 2; 5) receiving room. The sound transmission from a room, through a plate, to the cavity, or vice versa, is given by the resonant transmission, where the sound field excites the plate resonances which in turn radiate into the cavity. It is also given by the mass law, defined by the response of the non-resonant, mass-impeded, modes of the plate. This latter transmission path does not exist above the coincidence frequency.

Craik and Smith [8] present measurements and calculation of the sound transmission through double wall partitions. The walls studied are made of plasterboard, and in one case of chipboard, which are joined by wooden frames; material and geometrical data are given in [8]. Craik and Smith base their SEA model on the Price and Crocker formulation while they add plate-plate couplings via the frames. These frames are 3 m long and are modelled as Euler beams. They are erected in the vertical direction at 40 cm cc distance and the plates are nailed to the frames at 30 cm distance. The frame-plate transmission is modelled as point coupling.

Figures 1 shows measured and calculated sound reduction indexes for a double wall with a cavity depth of 100 mm; also 50 mm and 150 mm cavities are considered. The double wall resonances occur at 123 Hz, 87 Hz and 66 Hz, respectively, while the cavity depth equals half an acoustic wave length at 3.4 kHz, 1.7 kHz and 1.1 kHz. The coincidence frequency is in the 3.15 kHz third octave band. As can be seen, the Price and Crocker model, as interpreted by Craik and Smith, makes a god job at lower frequencies where the partition is modelled as one single plate; perhaps a bit surprising as the mode count is rather low at these frequencies. The model is also very god at higher frequencies when the cavity depth is not too small compared to the acoustic wave length. At intermediate frequencies, there are consistent errors, which increase with decreasing frequency. (These errors were largely reduced in Reference [3].)

The double wall resonance is not seen in the measured sound reduction index (SRI) while there is a plateau, extending approximately an octave above this frequency. The dip in the 160 Hz band, seen for all three cavity depths, might be caused by an increase in the plate mobility as in this band the distance between the frames is a bit more than half a flexural wave length in the plate so there are many plate resonances in this band. After the plateau, the reduction index starts increasing by some 9 dB per octave. Similar characteristics have been observed by the current author for other building structures as well as trimmed vehicle and aircraft structures. In the next sub section, the wave motion in a double wall structures are investigated in more detail to gain understanding of the vibroacoustic motion of double walls.





Left, Figure 1. Sound Reduction of building construction. Measurement and old calculation results from [8]. Right, Figure 2. Dispersion curves; blue dots, Waveguide FE; black line, uncoupled plates, flexural waves at top and longitudinal waves at bottom; red dashed line, plane acoustic wave.

2 Wave motion in double walls

The waveguide Finite Element (FE) method is a versatile tool for the investigation of wave motion in structures that have constant material and geometrical properties along one direction [6][9]-[10]. Using this method, the motion's dependence of the cross-sectional coordinates is approximated with FE polynomial shape functions, upon which follows a set of coupled ordinary differential equations describing wave propagation and decay along the structure. The plaster boards are described by quadratic deep shell elements and the fluid by quadratic Lagrange type elements [5].

Figure 2 shows the dispersion curves for the 100 mm double wall. The two straight curves at the bottom are the quasi longitudinal waves in the two plates. These have slightly different wave numbers as the plate have slightly different material properties. The two straight lines at the top are the flexural waves of the plate. Perhaps surprisingly, the flexural waves' wavenumbers are close to those for the uncoupled plates, also at frequencies below the double wall resonance. This is so because the fluid mass loading is low compared to the plates' mass and because the fluid cannot transfer any shear stress. The wave forms, however, exhibit coupling between the plates and the plates and the fluid, as can be seen in Figs 3. At low frequencies, one of the flexural waves is anti-symmetric and the other is symmetric while the plate amplitudes are roughly equal. Above the double wall resonance, the waves are localised to either of the plates while the amplitude of the other plate decreases rapidly with frequency.

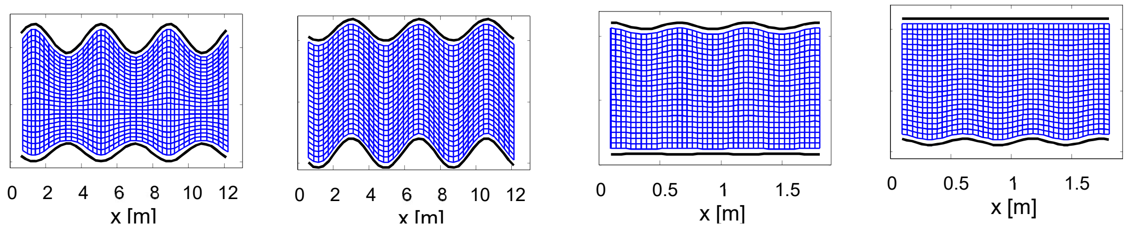


Figure 3. Plate waves. First two figures, about 3Hz; Last two figures, about 120 Hz

The waves that 'cut-on' (start propagating) at 87 Hz and 1.7 kHz are predominantly fluid waves that as frequency increases approach the dispersion curve for free acoustic waves. The wave form for the first of the predominantly fluid waves is shown in Fig 4. For all frequencies, the fluid motion is almost plane with some motion of the walls, which decreases with frequency. Most important for the formulation of a new SEA element: the fluid wave does not exist below the double wall resonance and it is supersonic, so the associated wall motion radiates well.

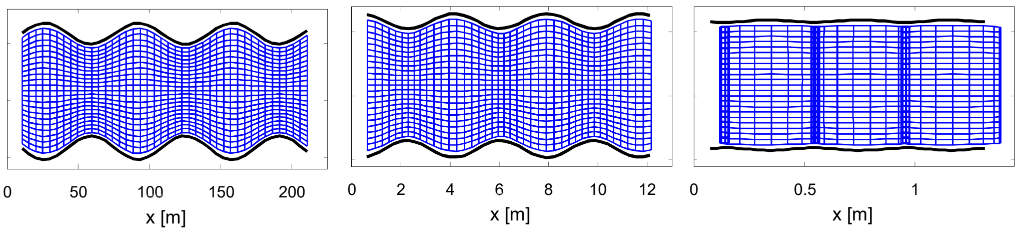


Figure 4 Wave form of Cavity-Wall wave. Left, 87 Hz; Middle 126 Hz; Right 840 Hz.

3 A new double-wall element

The new double wall formulation is illustrated in Figure 5. It is similar to Craik and Smith's model [8] except for that the cavity element is subdivided into two elements, and these do not describe only the fluid motion but also the mass impeded motion of the walls. One element models the almost plane motion of the cavity-wall and another element describe the oblique cavity modes.

3.1 Modal density

The waveguide FEM describes most accurately and conveniently the double wall wave motion. It is, however, also possible to describe the waves analytically in a lower frequency domain where the wall motion is mass impeded, as is detailed in Reference [5]. The wave solutions within the cavity are approximately given by

$$\tilde{p} = \tilde{p}_0 \left(\sin \gamma_r z + \mu_2 \, \gamma_r d \sin \gamma_r z \right) \mathrm{e}^{\mathrm{i}\kappa_r x} , \tag{2}$$

$$\kappa_r \left(\omega \right) = \mathrm{Re}\left(\sqrt{k_a^2 - \gamma_r^2} \right) . \tag{3}$$

The γ_r are solutions to a transcendental eigenvalue problem, which for quite heavy double walls, as those considered here, have solutions that are well approximated by

$$\gamma_0 = 2\pi f_{dw} / c ; \quad \gamma_r = r\pi / d , \; r = 1, 2, \ldots , \tag{4}$$

For each of these eigenvalues, there is a two-dimensional motion in the *x*-*y* plane. The number of modes below a frequency ω is given by

$$N\left(\omega \right) = S \, \kappa_r^2 / 4\pi . \tag{5}$$

It follows, the average modal density for the plane cavity-wall modes, in a band of width $\Delta\omega = \omega_u - \omega_l$, is given by

$$n_{cw} = S\left(\kappa_0^2 \left(\omega_u \right) - \kappa_r^2 \left(\omega_l \right) \right) / 4\pi \, \Delta\omega . \tag{6}$$

3.2 Coupling losses

The power radiated from the resonant vibrations of one of the walls to a semi-infinite room is

$$P_{rad} = \rho_o c\, S\, \sigma^{(p,r)} \left\langle \tilde{v}^2 \right\rangle = \rho_o c\, n_p\, \hat{e}_p\, \sigma^{(p,r)} \big/ m_p \qquad (7)$$

where n_p, $\hat{e}_p$ and m_p are the modal density, modal power and the mass per u.a. of the wall and $\sigma^{(p,r)}$ is the radiation efficiency from a plate to an acoustic volume. It is calculated by Leppington's formula [12] and is a function of the plate dimensions and the ratio of the wavenumbers in the plate and in the receiving acoustic volume. The multipliers suggested in Fig. 5 of reference [8] are applied. Also, at frequencies below the coincidence frequency, there is point radiations at the mounting of the walls to the frames.

Thus, the coupling losses for the sound radiation from plates to the rooms and to the cavity are given by

$$C^{(p,r)} = \omega\, n_p\, \eta_c^{(p,r)} = P_{rad} \big/ \hat{e}_p = \rho_o c\, n_p\, \sigma^{(p,r)} \big/ m_p\ , \qquad (8)$$

To evaluate the coupling losses for the energy flow from the cavity-wall modes to a room, the wall's mean square vibration velocity need be related to the modal power, which is made based on Eq (2), as detailed in Reference [5]. These waves are, as seen in Figure 1, supersonic, approaching sonic speed as frequency increases; so, their radiation efficiency is large.

The structural point couplings between plates and frames are modelled as in Ref. [8] except for that the frames are modelled as Timoshenko beams.

4 Comparison with measurements

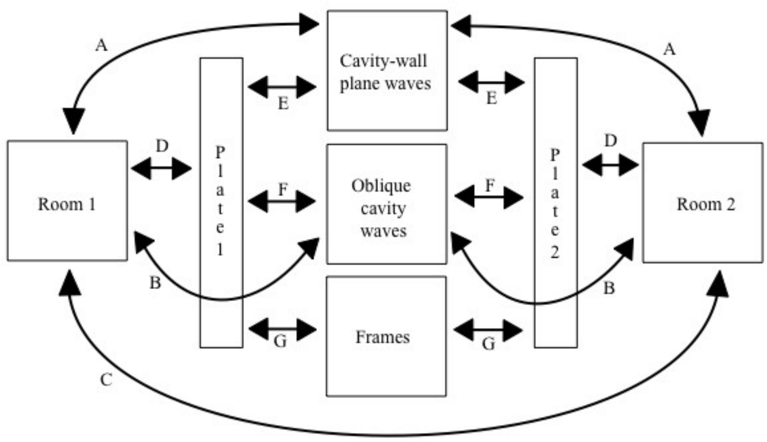


Figure 5. Proposed SEA for a double wall. A, mass impeded plane cavity-wall element - room path below the coincidence frequency; B, oblique cavity-wall element - room path below the coincidence frequency; C, room to room non resonant path described by the mass-law below the double wall frequency; D, room-plate path calculated by Leppington's radiation efficiency plus, below the coincidence frequency, point radiation at the plate mounting; E and F, as C and D; **G**, plate - frame path

A particularly difficult problem in both dynamic and energy-based modelling of vibro-acoustic response of real life structures is the estimation of damping. Here, the frequency independent loss factors reported in [8] for the plates and frames of $\eta = 0.01$ and $\eta = 0.015$ are adopted. The equivalent absorption areas of the rooms are chosen so large that their precise values do not influence the calculated SRI. The reverberation time for the 100 mm cavity, with no extra damping, is reported in Fig 7 of reference [8]. The cavity damping loss factors are derived, equally for the plane cavity-wall modes and the oblique cavity modes, from these reverberation times. The damping loss factors for the 50 mm and the 150 mm cavities are assumed to be given by the one for the 100 mm cavity in inverse proportion to the cavity depth,

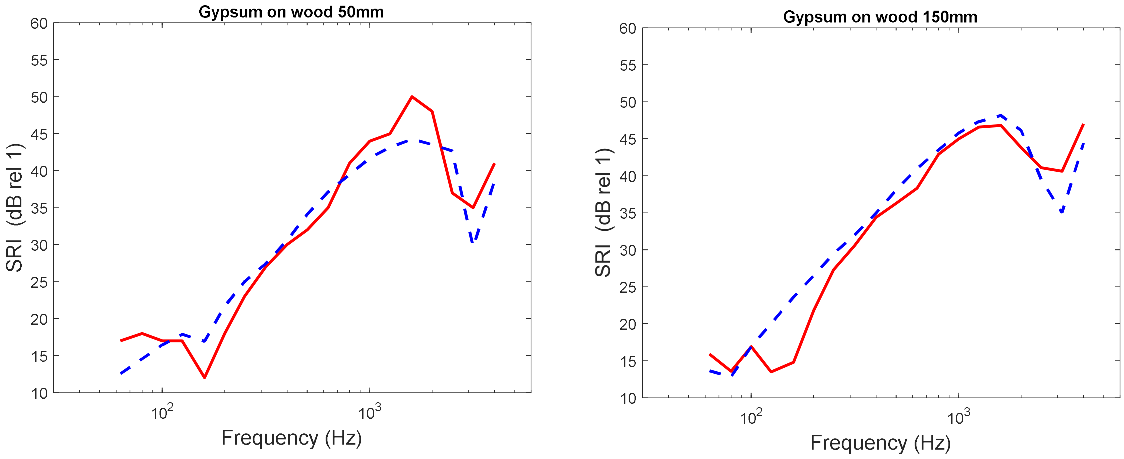


Figure 6 SRI of 50 mm and 150 mm building construction. Solid line, measurements [8]; dashed line, SEA model

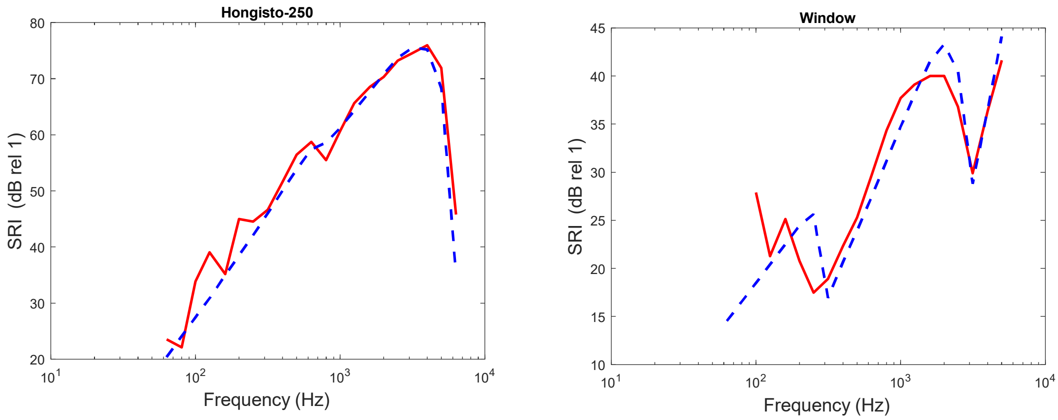


Figure 7. SRI. Left, 250 mm cavity and steel walls [2]. Right, 12 mm deep glass window [1]

since, it is believed, most of the damping arises, because of viscosity and heat conduction, at the walls and frames and, because of air pumping, at the interface of these structural components. This surmise is supported by that the measured loss factor decays with the square root of frequency.

Calculated SRIs are shown in Figure 6 together the measured values of Craik and Smith. The new model underestimates the SRI at the coincidence frequency, in the 3.15 kHz band, by some 4-8 dB, which is unexplained, as the new and old models should be similar at such high frequencies. The agreement in the low frequency region, below, around and an octave above the double wall resonance, is good but somewhat erratic, probably so as the mode count is rather low for most of the elements. Additionally, there is a consistent overestimation in the 160 Hz band. Other than this, the agreement between calculations and measurements is excellent for the 50 mm and the 100 mm cavities and good for the 150 mm cavity. Given the expected uncertainty of an SEA model in general, and the damping estimates, the results are good.

Figures 7 compares calculations and measurements of the SRI for one deep cavity with steel walls in which the cut-on of the oblique cavity waves is seen around 600 Hz [2]. Also, results for a glass window, describe in Ref. [1], are found in Figures 7. The cavity damping for these structures has been adopted from the measurements and are given as inversely proportional to the square root of frequency, like the measured damping in Ref. [8]

5 Summary

An analysis of the wave motion within double wall structures shows that the plane acoustic waves that cut-on at the double-wall resonance involve quite large, mass impeded, motion of the walls. The modes that correspond to these waves define one SEA element for the cavity; the other element is defined by the oblique cavity waves that cut-on when the cavity depth approximately equals half an acoustic wave length. This element formulation obsoletes the non-resonant transmission paths used in earlier double-wall formulations [7]-[8]. Instead there is a direct coupling between the acoustic volumes and the resonant cavity-wall modes. As already noted by Craik [3] the plane cavity-wall waves are supersonic and have radiation efficiency greater than unity, which was the value used in earlier works for the non-resonant transmission. The new element attributes the frequency dependent radiation efficiency and the varying modal density and the frequency dependent relation between modal power and wall motion. This new formulation explains why the reduction index does not have a distinct minimum at the double-wall resonance but instead have a rather constant value in a frequency region, which extends roughly an octave above this resonance. This example emphasises that SEA elements are elements of response that need not be localised to substructures of the whole structure. The successful application of SEA to a new kind of structure, therefore, requires diagnostic measurements and diagnostic calculations so that the proper elements can be identified.

The new double-wall formulation provides results that agree quite well with Craik and Smith's measurements [[8]] of three constructions with gypsum boards. Also, a fair agreement is found for the deep cavity with steel walls in Ref [2] and the shallow cavity with glass walls in Ref. [1].

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References

[1] M Villot, C Guigou and L Gagliardini. Predicting the acoustical radiation of finite siz multi-layered structures by applying spatial windowing on infinite structures. *Journal of Sound and Vibration* **245** (2001) 433-455.

[2] V. Hongisto, M. Lindgren, R. Helenius, Sound insulation of double walls – an experimental parameter study, *Acta Acustica united with Acustica* **88** (2002) 904-923.

[3] R.J.M. Craik, Non-resonant sound transmission through double walls using statistical energy analysis*, Applied Acoustics* **64** (2003) 325-341.

[4] J Poblet-Puig, A Rodriguez-Ferran, C Guigou-Carter and M Villot. The Role of Studs in the Sound Transmission of Double Walls *Acta Acustica united with Acustica* **95** (2009) 555-567.

[5] Mathias Barbagallo. *PhD Thesis KTH TRITA-AVE 2013:04* "Statistical Energy Analysis and variational principles for the prediction of sound transmission in multi-layered structures"

[6] Hao Liu *PhD Thesis KTH TRITA-AVE 2014:51* "Wave modelling techniques for medium and high frequency vibroacoustic analysis including porous materials"

[7] A.J. Price, M.J. Crocker, Sound transmission through double panels using statistical energy analysis, *JASA* **47** (1970) 683-693.

[8] R.J.M. Craik, R.S. Smith, Sound transmission through double leaf lightweight partitions, part I: airborne sound transmission, *Applied Acoustics* **61** (2000) 223-245.

[9] C-M. Nilsson, S. Finnveden, Waves in thin-walled fluid-filled ducts with arbitrary cross-section, *Journal of Sound and Vibration* **310** (2008) 58-76.

[10] S. Finnveden, M. Fraggstedt, Waveguide finite elements for curved structures, *Journal of Sound and Vibration* **312** (2008) 644-671.

[11] Craik, R.J.M., Sound transmission through buildings using statistical Energy Analysis, Gower press, 1996.

[12] F. G. Leppington, E.G. Broadbent, K.H. Heron, The acoustic radiation efficiency of rectangular panels, *Proceedings of the Royal Society London A* **382** (1982) 245-271.