The effect of high level acoustic excitation on the acoustic properties of perforates as used in mufflers and acoustic liners.

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Perforates are used for noise control in automotive mufflers and aircraft engine liners as well as for other vehicles and machines. Their acoustic properties and noise reduction are known to depend on the mean flow field and other parameters such as temperature and acoustic excitation level. It is therefore of interest to understand how the properties of perforates varies with the level of acoustic excitation. This paper gives an overview of high level nonlinear effects on the acoustic properties of perforates. It includes semi-empirical models as well as experimental studies. Methods for studying nonlinear effects and harmonic interaction effects, for perforates, using single tone excitation and Poly-harmonic distortion models or nonlinear scattering matrices are discussed. These techniques typically require measurements with a number of different acoustic loads. It would be more attractive to directly be able to extract the nonlinear acoustic properties from a more limited set of experiments using either random or periodic excitation. Multi input – single output techniques for nonlinear system identification using broadband random excitation has been tried with limited success. One reason is the mixing of the sound pressure signal incident from the acoustic source with the sound pressure transferred to higher frequencies by nonlinear effects at the perforate sample. The present paper includes an attempt to combine band-limited broadband excitation with Poly-harmonic distortion models or nonlinear scattering matrices describing the nonlinear transfer of energy to higher frequencies.

1   Introduction

Perforates are used for noise control in automotive mufflers and aircraft engine liners as well as for other vehicles and machines. Their acoustic properties and noise reduction are known to depend on the mean flow field and other parameters such as temperature and acoustic excitation level. It is therefore of interest to understand how the properties of perforates varies with the level of acoustic excitation. In a number of papers experimental techniques for determining acoustic impedance and two-port data for perforates under non-linear conditions have been developed [1-7]. Experiments were made using both pure tone and random excitation and the relevant parameters controlling the non-linearity were discussed. In [1] a study of harmonic interaction effects using two-tone excitations was made and in [2] the study was extended to multi-tone excitation for different types of perforates. In [3] a study was made of using non-linear system identification techniques for this purpose. The effect of sample non-linearity when performing impedance tube measurements were studied in [4] along with an outline of multi-port techniques for characterization of samples with non-linear properties. These multi-port techniques were further developed and experimentally tested in [5-7]. Many investigations of nonlinear effects occurring when high amplitude sound waves are incident on perforated plates or orifice plates have been published, see e.g., [8-16]. It is generally agreed that the non-linear losses are associated with

vortex shedding at the outlet side of the orifice or perforate openings. The nonlinear multi-port techniques with sinusoidal excitation developed and tested in [4-7] aimed at taking non-linear energy transfer between sound field harmonics into account. The methods for studying nonlinear harmonic interaction effects, for perforates, using single tone excitation [4-7] typically require measurements with a number of different acoustic loads. It would be more attractive to directly be able to extract the nonlinear acoustic properties from a more limited set of experiments using either random or periodic excitation. In [17] multi input – single output techniques [18-22] for nonlinear system identification were tried without much success. The idea of treating a nonlinear path as a separate non-linear input after which system identification is performed as for a linear two input one output system was first introduced by Bendat [18-20]. The general methodology, for arbitrary nonlinear systems, as used in this paper was first published by Rice and Fitzpatrick [21]. The techniques have later been summarized by Bendat [20]. An example of a more recent work applying a modified version of the technique to mechanical system is [22].

In the present paper a method for extracting nonlinear scattering matrix data according to [5-6] but using broadband random excitation is attempted. The background is an impedance tube test from [6] comparing impedance results obtained when there was random noise excitation in the whole frequency range up to 2000 Hz with results obtained if there was only excitation up to 500 Hz. Figure 1 shows the real part of the normalized impedance, where the black solid line curves were obtained using broadband excitation covering the whole frequency range while the red dashed line curves were obtained with excitation only up to 500 Hz. The explanation for the results obtained at high levels of excitation is that transfer of energy to higher frequencies gives a sound source located at the sample surface. The sound field from this source will interact with the sound field from the loudspeaker source. The two-microphone wave decomposition technique used in impedance tube measurements [23-26] assumes that we know where the source is located relative to the microphones and the sample under test. If the sound source is located at the sample, but we assume that the sound source is at the loudspeaker side, we will as discussed in [6] get a result corresponding to the impedance looking back into the impedance tube from the sample but with the wrong sign. It can be seen that due to the non-linear energy transfer to higher frequencies a result is obtained also in the frequency range where there is no excitation. If you change the sign this results look like what you expect to see for an impedance measurement in an open ended pipe. With random excitation in the whole frequency range we get a mix of the impedance caused by the direct excitation from the loudspeaker and the impedance results caused by the non-linear energy transfer giving a sound source at the sample. It is therefore not possible to get a result characterizing the non-linear sample using random excitation. It can also be noted from Figure 1 that the resistance for the case with excitation up to 500 Hz becomes negative at around 800 Hz indicating that the source is at the sample, giving sound generation instead of dissipation. The explanation is that the level of excitation caused by non-linear energy transfer increases with increasing frequency because of an accumulation effect where more frequency components from below 500 Hz can contribute to a specific frequency component as the frequency increases.
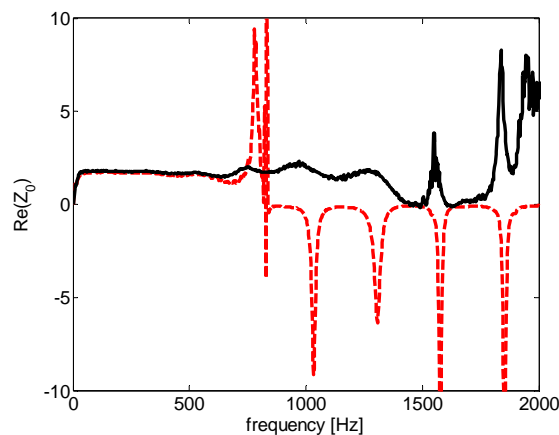

Figure 1: Real part of normalized impedance for a perforate sample with 2% porosity, 1 mm hole diameter and 2 mm thickness with 116 dB sound pressure level and 0.012 m/s particle velocity at the sample: ― ― ―broadband random excitation up to 2000 Hz, ――――broadband random excitation up to 500 Hz.

The methods developed in [4-7] for single tone excitation are based on the assumption, backed by experimental evidence, that nonlinear energy transfer at the sample mainly occurs from lower to higher frequencies and to odd

harmonics of the excitation frequency. The strongest coupling is between the excitation frequency and the third harmonic. The idea tested in the present paper is that if bandlimited random excitation is used, for instance excitation only below 500 Hz such as in Figure 1 this information can be used to study nonlinear energy transfer to frequencies three times as high. For instance using broadband excitation at 300 Hz and studying the transfer of energy to 900 Hz, assuming that the only signal occurring at 900 Hz is caused by nonlinear energy transfer at the sample.

2   Nonlinear system identification

Using the techniques described in [6] it is possible to study harmonic interaction effects. Figure 2 shows a sketch of the sample mounted at the end of an impedance tube. A whole series of assumptions and simplifications need to be made to apply the techniques from [6] to the data presented here. It is assumed that:

- The signals are analytical.
- Nonlinear energy transfer only occurs from lower frequency components to higher frequency harmonics.
- The nonlinear energy transfer is only to odd harmonics.
- There is only one frequency component with high level excitation and the system components for other frequencies can be determined from linear scattering matrix or reflection coefficient measurements.

With these assumptions and simplifications the relation between the high level excitation at frequency f and response at f and $3f$ can be described by the following matrix equations for a one-port, such as in the impedance tube,

$$\begin{pmatrix} P_r(f) \\ P_r(3f) \end{pmatrix} = \begin{bmatrix} S_{f,f} & 0 \\ S_{3f,f} & S_{3f,3f} \end{bmatrix} \begin{pmatrix} P_i(f) \\ P_i(3f) \end{pmatrix} \quad , \tag{1}$$

Where: P_i and P_r are the incident and reflected waves (towards and away from the sample) and $S_{3f,3f}$ is measured through a separate low level excitation measurement at frequency $3f$ while the other two components are ´measured with varying level of excitation at frequency f.
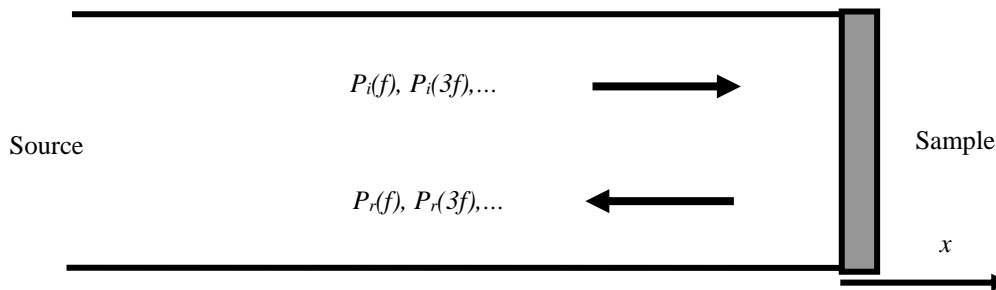

Figure 2: Pressure waves in a test duct terminated by a non-linear sample.

There is only excitation at frequency f so the pressure waves travelling in positive x-direction at higher frequencies are caused by reflection at the source.

3   Experimental technique and setup

An impedance tube was in the experimental tests used to perform the wave decomposition and obtain the pressure wave amplitudes P_i and P_r . Three B&k ¼-inch microphones were used giving the smallest microphone separation 5 cm and the largest 30 cm. This made it possible to cover a frequency range from 60 Hz up to the maximum frequency used in the experiment 2000 Hz [25,26]. The sample was placed in a holder at the end of the duct and measurements were made with and without the sample. Time domain data was collected using random excitation. The measured pressures were Fourier transformed and the pressure and particle velocity was calculated at the sample cross section assuming linear plane wave propagation in the duct. Tests were made for a number of different perforate samples with varying porosity, hole diameter and hole thickness. The results presented here are for a sample with 2% porosity, 1 mm hole diameter and 2 mm thickness.

4   Experimental results and discussion

Measurements were made with 10 different levels of excitation and either with broadband random excitation up to 2000 Hz or up to 500 Hz. In both cases data was taken with a sampling frequency of 5120 Hz giving a maximum analysis frequency of 2000 Hz. In the analysis broadband random data covering a frequency range from f = 265 Hz such that $3f$ = 795 Hz, to f =500 Hz such that 3f = 1500 Hz, was used. Data presented here were determined with analysis bandwidths Δf = 2.5 Hz which gives results for 95 frequencies. To illustrate the nonlinearity data will be presented either as a function of the absolute value of the amplitude of the incident pressure wave at the sample (Abs($P_i(f)$)) or as a function of an inverse Strouhal number (St) proportional to the particle velocity ($u(f)$) at the sample

$$1/St = u(f)/\omega t \qquad (2)$$

where ω is the angular and t the thickness of the perforate. The inverse Stouhal number is equivalent to the ratio between particle displacement and perforate thickness. In some cases results will also be presented as function of frequency.

Figure 3 shows the reflection coefficient S_{ff} at the excitation frequency, for all 95 frequencies, plotted against the inverse Strouhal number. The nonlinearity can be clearly seen since the result varies with the level of excitation. In Fig. 3 c) and d) results are presented as differences compared to the lowest level of excitation in attempt to remove the linear part and only study the nonlinear variation. It can be seen that this gives a reasonable collapse of the phase data while the variation is still large in the absolute values. Figure 4 shows the same results but plotted against the absolute value of the incident pressure wave. It can be seen that this way of plotting the data gives a better collapse of the absolute values in Fig. 4 c) while the phase data in Fig. 4 d) now shows a larger variation.

Figure 5 shows the reflection coefficient S_{3f3f} as used in Eq. (1) as a function of frequency. It is evaluated for cases with excitation up to 2000 Hz and only for the lowest level of excitation representing the linear case. It can be seen that it has only a moderate variation with frequency.

Figure 6 shows the absolute value of S_{3ff} from Eq. (1) which is related to the transfer of energy from the fundamental frequency to the third harmonic. It can be seen that there is a scatter in these results both when plotted against inverse Strouhal number and absolute value of the incident pressure wave amplitude.

In Figure 7 a subset of data with frequencies between 300 and 350 Hz were used to produce an average for S_{3ff}, which is compared to a result obtained using single tone excitation at 220 Hz. There is a reasonable agreement between the two results. To further investigate if the "model" for S_{3ff} produced from broadband excitation can be used to predict results for other cases a comparison is made between pressure wave amplitude $P_r(3f)$ obtained using single tone excitation at 220 Hz and predicted using the measured $P_i(3f)$ and $P_i(f)$ and S_{31} and S_{33} from the model determined using broadband random excitation. The result is shown in Fig. 8 and gives a good agreement for the absolute value and a reasonable agreement for the phase. The conclusion from this study is that the method of determining nonlinear scattering matrix data from bandlimited broadband random excitation instead of from single tone excitation shows promising results.
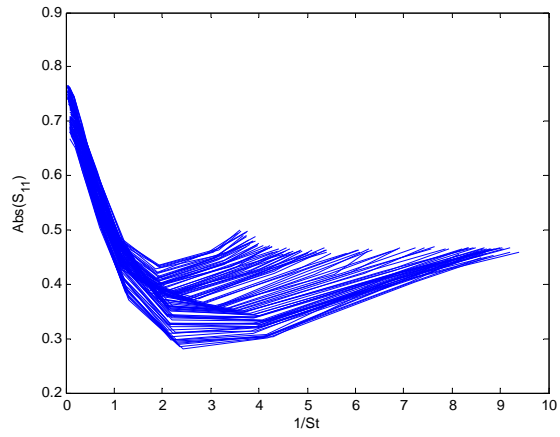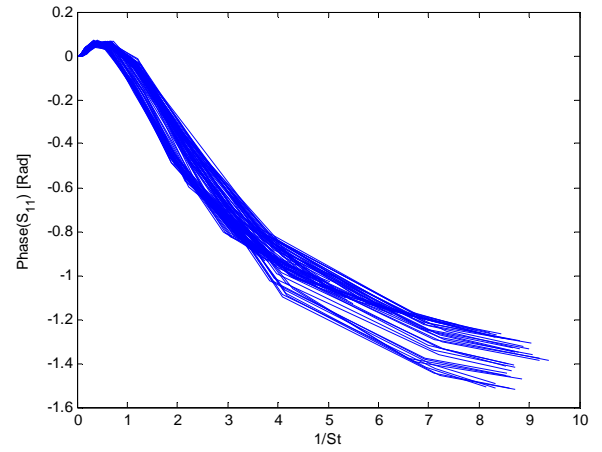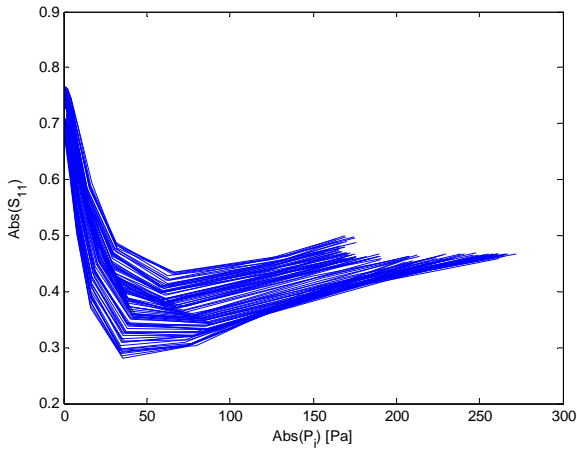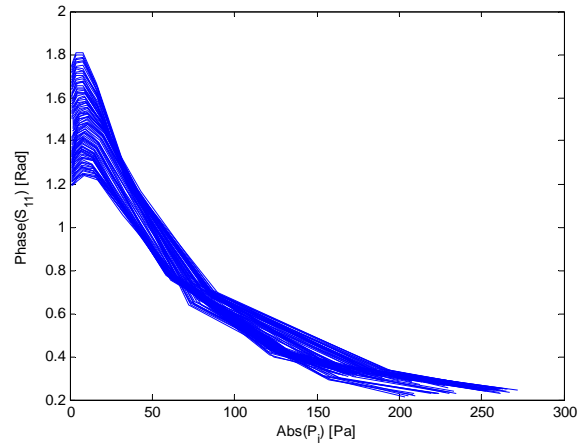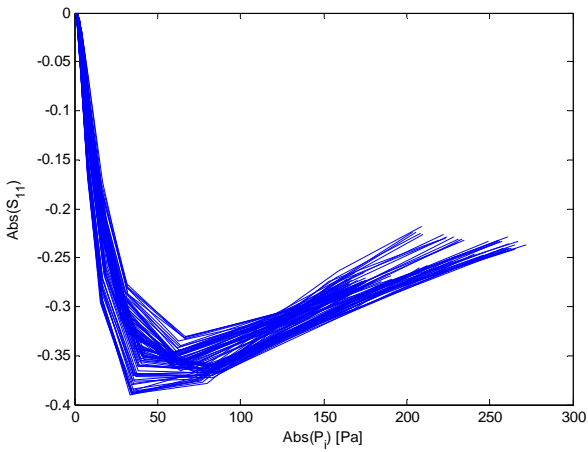
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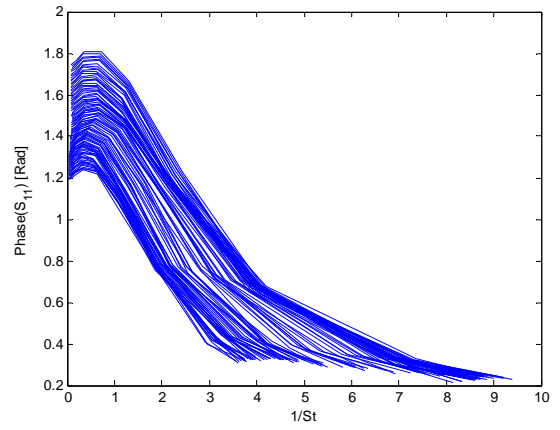
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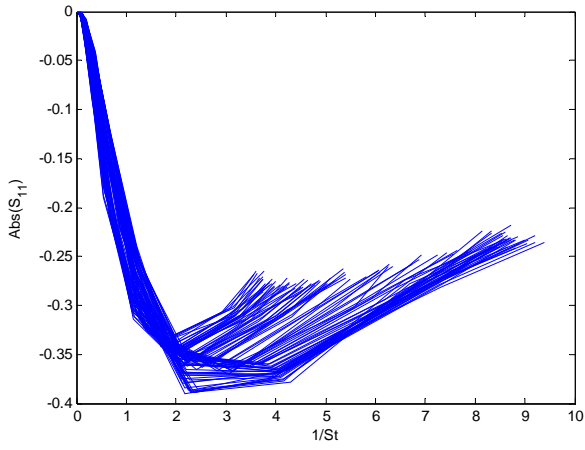
c)

d)

Figure 3: Reflection coefficient S_{ff} according to Eq. (1) as function of inverse Strouhal number. a) Absolute value, b) phase, c) variation in absolute value compared to the lowest level of excitation, d) variation in phase compared to the lowest level of excitation.
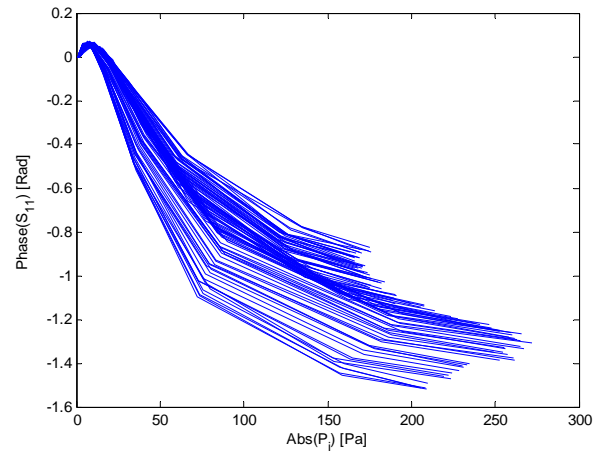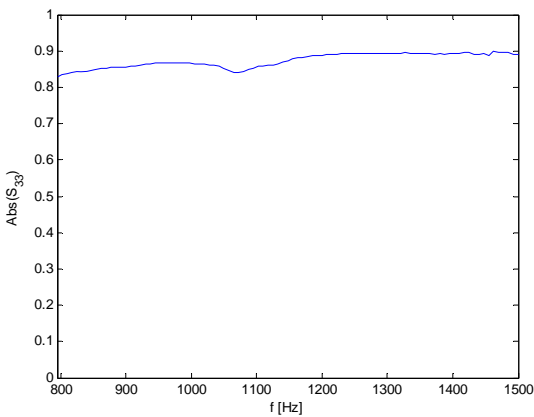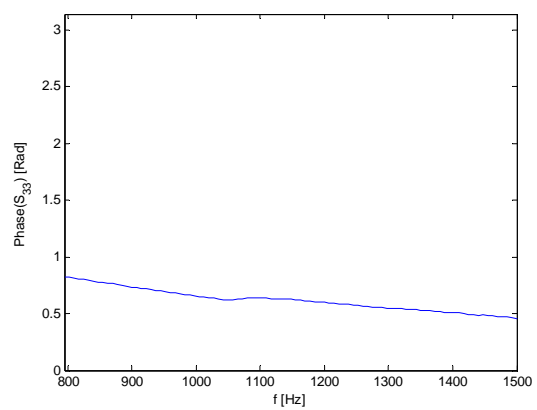
a)

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Figure 4: Reflection coefficient $S_{f,f}$ according to Eq. (1) as function of inverse absolute value of the incident pressure wave amplitude. a) Absolute value, b) phase, c) variation in absolute value compared to the lowest level of excitation, d) variation in phase compared to the lowest level of excitation.
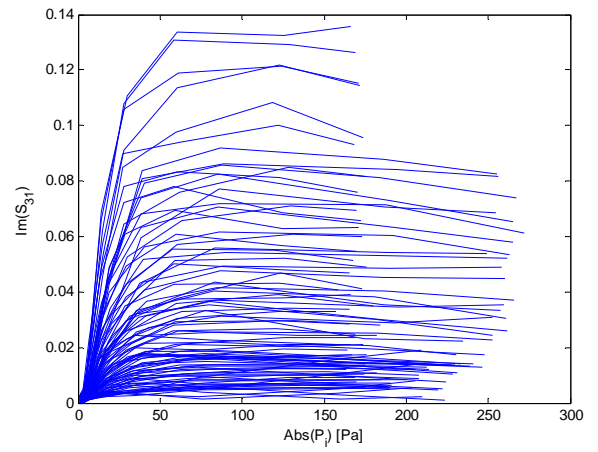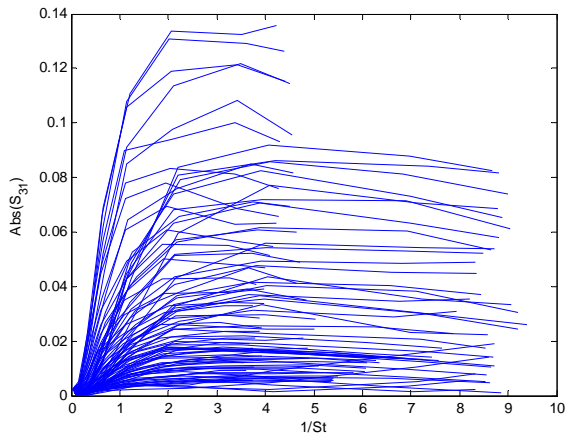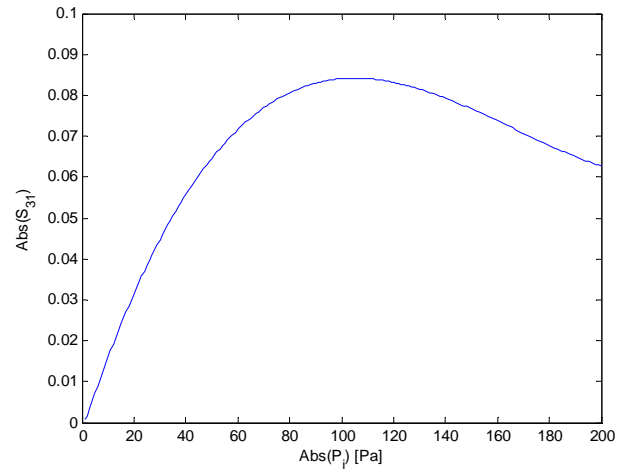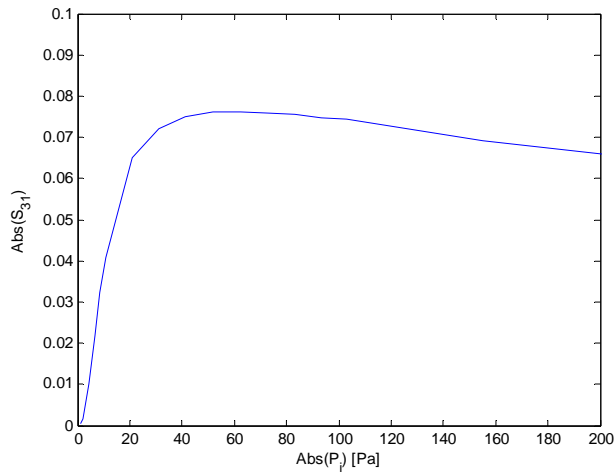

a)

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Figure 5: Reflection coefficient $S_{3f,3f}$ according to Eq. (1) as function of frequency. a) Absolute value, b) phase.
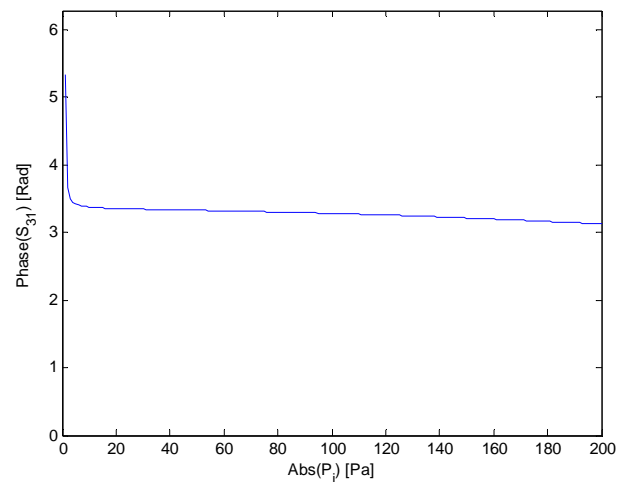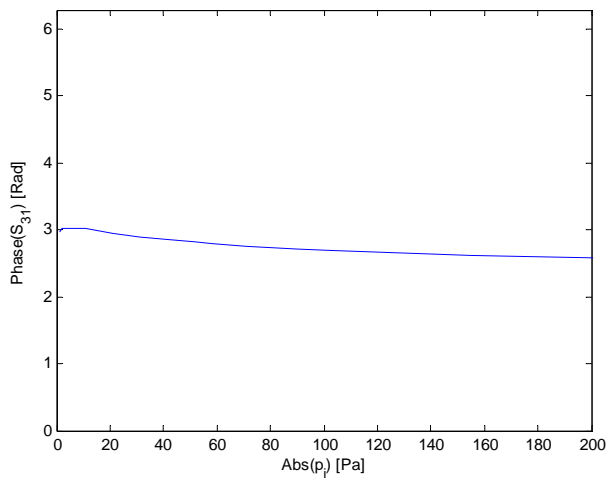
a)

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Figure 6: Absolute value of $S_{3f,f}$ according to Eq. (1). a) Plotted against inverse Strouhal number, b) plotted against absolute value of the incident pressure wave.
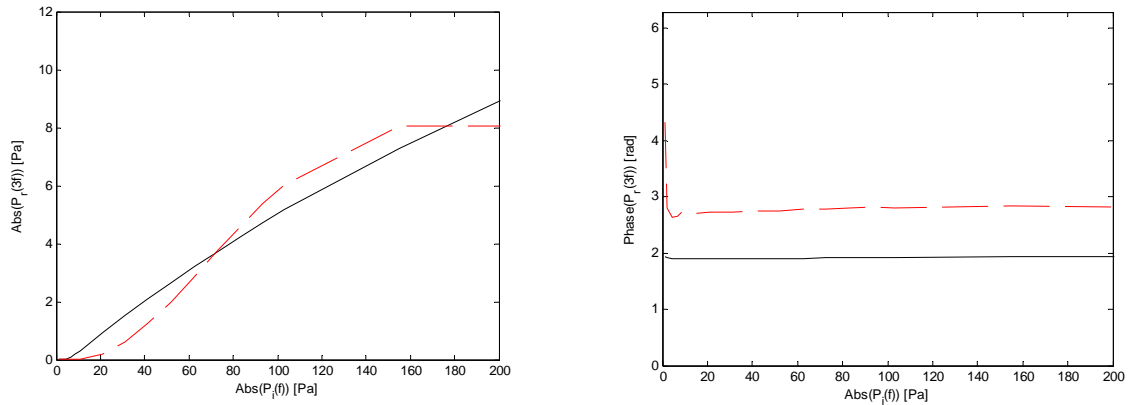

a)

b)

c)

d)

Figure 7: Nonlinear scattering matrix element $S_{3f,f}$ according to Eq. (1). a) Absolute value determined from single tone excitation at 220 Hz, b) absolute value determined from broadband random excitation, c) phase determined from single tone excitation at 220 Hz, d) phase determined from broadband random excitation.

a)   b)

Figure 8: Pressure wave amplitude at 660 Hz for pure tone excitation at 220 Hz, black solid line – measured, red dashed line – predicted using model from Eq. (1). a) Absolute value, b) phase.

5   Conclusions

Experimental methods for determining nonlinear acoustic properties of perforates used in automotive mufflers and aircraft engine liners have been discussed. A new idea where nonlinear scattering matrix data, which has previously only been obtained with tonal excitation, is measured using bandlimited broadband random excitation has been tested. The results are promising.

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