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Simplified models for acoustic screens

Mathieu Gaborit
KTH Royal Institute of Technology, Stockholm, Sweden, **gaborit@kth.se**
LAUM UMR CNRS 6613, Le Mans Université, Le Mans, France

Peter Göransson
KTH Royal Institute of Technology, Stockholm, Sweden

Olivier Dazel
LAUM UMR CNRS 6613, Le Mans Université, Le Mans, France

Abstract

In order to protect an acoustic panel and to improve its efficiency, thin films or screens can be laid on its free surfaces. This technique is widely used in the industry and, because of their strong effect, the added coatings must be included in the simulations from the beginning of the design stage. Many films and screens being porous, they require the use of either a complete Biot model for the simulation or at least an equivalent fluid one. On the other hand, the properties of such coatings tend to be difficult to characterise which is due to their mechanical properties and small thickness (less than a millimeter). Two questions then arise: how to efficiently account for uncertainties on the parameters in a simulation ; and is it possible to derive a model which is less sensitive to measurement errors?

To answer those two questions, the present work focuses on a simple, yet accurate, model for resistive films used in acoustic multilayer absorbers. The key idea is to reduce the number of parameters needed to correctly account for the effects of the screen. The benefit of the proposed approach are twofold. First, modelling the impact of uncertainties is rendered easier by reducing the amount of random draws in the parameters' space. Secondly, having only a small number of parameters to characterise is a step towards better reproducibility and better precision.

Using asymptotic expansions and performing a dimensional analysis simplifications of the model are proposed. These are then tested by introducing them in a transfer matrix model and their impact of the accuracy is studied. Particularly, the contribution of each wave to the different fields is evaluated in order to drive the simplification process further.

1 Introduction

Noise has been identified for long as having a major impact of day to day life and wellness. In the recent years, much effort has been devoted to the design of noise mitigation systems, both in industry and academia. While some teams focus on new approaches such as metamaterials [1, 2, 3] others aim at improving existing systems. One of the most widely adopted designs for acoustic absorbers relies on stacked layers of poroelastic materials (foam, fiber panels,...). This design received lately a lot of attention from the community with two main goals: to better understand the governing phenomena [4, 5] and to optimise the system properties to achieve given performance.

The present work focuses on a particular element of these multilayer panels: coatings. Indeed, to protect the panel and for aesthetic reasons, the multilayer absorbers commonly carry thin permeable films on their free surface(s). Despite their small thickness (generally between 0.1 and 1mm) these screens have a noticeable impact on the acoustic behaviour and must thus be considered. To this date, simulation of acoustic screen of films used either an equivalent fluid approach or a complete Biot-Johnson-Champoux-Allard (Biot-JCA) model. On one hand, the former tends to artificially smooth the acoustic response, missing some of the resonances and on the other hand, the latter is a quite heavy model designed to account for complex interphase coupling that seems unreasonable (unphysical?) in such thin layers.

The present work aims at proposing a simpler model accounting for the resonant effects while reducing the number of physical parameters needed. It is intended as a replacement for the more elaborate approaches used in the current Transfer Matrix Models. In order to derive this model, it is proposed to linearize the propagation using a first order expansion of the exponentials and perform a dimensional analysis to filter out the terms linked to small effects (section 2). The approach is validated using numerical simulations and which are compared to the results to a full Biot-JCA model for a large number of films (section 3). Note that throughout this paper, a positive ($e^{j\omega t}$) convention is used.

2 Derivation of the simplified model

The proposed approach relies on a transfer matrix approach to describe the evolution of the physical field through a layer of poroelastic material. The derivation of the matrix starts with the two motion equations and two constitutive law from the Biot model [6, 7]. In the present case, the equations are those of the strain-decoupled model [8] with the time-harmonic hypothesis:

$$\begin{aligned} \hat{\sigma}_{ij,j} = -\omega^2\tilde{\rho}_s u_i^s - \omega^2\tilde{\rho}_{eq}\tilde{\gamma}u_i^t, \qquad -p_{,i} = -\omega^2\tilde{\rho}_{eq}\tilde{\gamma}u_i^s - \omega^2\tilde{\rho}_{eq}u_i^t, \\ \hat{\sigma}_{ij} = \hat{A}\nabla\cdot\boldsymbol{u^s}\delta_{ij} + 2N\varepsilon_{ij}, \qquad p = -\tilde{K}_{eq}\nabla\cdot\boldsymbol{u^t}. \end{aligned} \tag{1}$$

In these equations, $\boldsymbol{u}^s$ and $\boldsymbol{u}^t$ are respectively the solid phase a total displacements fields, p is the interstitial pressure, $\hat{\sigma}$ and ε the in vacuo stress and strain tensors and δ_{ij} the Kronecker symbol. The other coefficients in Eq. (1) are physical parameters, i.e. $\tilde{\rho}_{s,eq}$ are densities, $\tilde{K}_{eq}$ a compressibility, $\hat{A}$ and N the Lamé coefficients and $\tilde{\gamma}$ a fluid/solid coupling term. A more thorough explanation about these equations is given in Ref. [8].

Considering a layer of infinite extent in the (x, y) plane and of thickness d excited by a plane wave travelling towards the positive values on the z axis, the transfer matrix is derived using the so-called Stroh formalism [9, 4, 10]. This approach allows to express the evolution of fields along the z direction using a set of first order partial differential equations on the state vector $\boldsymbol{s}$:

$$\frac{\partial \boldsymbol{s}(z)}{\partial z} = \boldsymbol{\alpha}\boldsymbol{s}(z), \qquad \text{with } \boldsymbol{s}(z) = \left\{\hat{\sigma}_{xz}(z),\ u_z^s(z),\ u_z^t(z),\ \hat{\sigma}_{zz}(z),\ p(z),\ u_x^s(z)\right\}^T. \tag{2}$$

An important remark is that not all the physical fields are present in the state vector. For instance, u_x^t and $\hat{\sigma}_{xx}$ are absent but they can be deduced from linear combinations of the state vector's components.

The $\boldsymbol{\alpha}$ state matrix is central to this approach and it's derivation has been addressed multiple times in the literature, for instance in[9] for this specific case (see Ref. [11, 10] for a more general discussion). In this work, the derivation of $\boldsymbol{\alpha}$ is omitted for conciseness and the one proposed in the appendix A.1 of Ref. [8] will be used:

$$\boldsymbol{\alpha} = \begin{bmatrix} 0 & 0 & 0 & jk_x\frac{\hat{A}}{\hat{P}} & jk_x\tilde{\gamma} & -\frac{\hat{A}^2-\hat{P}^2}{\hat{P}}k_x^2 - \tilde{\rho}\omega^2 \\ 0 & 0 & 0 & \frac{1}{\hat{P}} & 0 & jk_x\frac{\hat{A}}{\hat{P}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tilde{K}_{eq}} + \frac{k_x^2}{\tilde{\rho}_{eq}\omega^2} & -jk_x\tilde{\gamma} \\ jk_x & -\rho_s\omega^2 & -\tilde{\rho}_{eq}\tilde{\gamma}\omega^2 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_{eq}\tilde{\gamma}\omega^2 & \tilde{\rho}_{eq}\omega^2 & 0 & 0 & 0 \\ \frac{1}{N} & jk_x & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

where $\hat{P} = \hat{A} + 2N$.

As the medium is homogeneous, one may derive the transfer matrix $\boldsymbol{T}(d)$ associated to a layer of thickness d by solving Eq. (2) which leads to:

$$\boldsymbol{s}(0) = \boldsymbol{T}(d)\boldsymbol{s}(d),\ \boldsymbol{T}(d) = \exp\left(-d\boldsymbol{\alpha}\right). \tag{4}$$

The remaining part of this section focuses on simplifying the $\boldsymbol{T}(d)$ matrix using different hypotheses. The most important among these is that the coating layer is almost always very thin ($d \ll 1$), especially compared to the wavelengths of the waves in the medium ($kx \ll 1$, with k the wavenumber in the poroelastic medium or in air). Such a hypothesis allows for an approximation of the matrix exponential of Equation Eq. (4) by its first order Taylor expansion:

$$\boldsymbol{T}(d) \approx \boldsymbol{I} - d\boldsymbol{\alpha} + \mathcal{O}(d^2) \tag{5}$$

with $\boldsymbol{I}$ the identity matrix. An important remark is that this first approximation reduces the propagation through the film to a jump of the field values involved. Indeed, Equation Eq. (5) indicates that all the fields are transferred and altered by a combination of the other fields controlled by the state matrix $\boldsymbol{\alpha}$.

The next goal is to identify the most important contributions in $\boldsymbol{\alpha}$ and eliminate all others. To this end, the second proposed approximation is introduced, and concerns the transfer of solid displacement described by the second and last lines of $\boldsymbol{T}(d)$:

$$u_z^s(0) = u_z^s(d) + \frac{d}{\hat{P}}\hat{\sigma}_{zz}(d) + jk_x d\frac{\hat{A}}{\hat{P}}u_x^s(d), \qquad u_x^s(0) = u_x^s(d) + \frac{d}{N}\hat{\sigma}_{xz}(d) + jk_x d u_z^s(d) \tag{6}$$

The constitutive laws in Equation Eq. (1) allow for the following approximations for $\hat{\sigma}_{xz}$ and $\hat{\sigma}_{zz}$:

$$\hat{\sigma}_{zz} \approx \hat{P}jk_s u_z^s, \qquad \hat{\sigma}_{xz} \approx \hat{N}j(k_x u_z^s + k_s u_x^s). \tag{7}$$

where k_s corresponds to the wavenumber of the leading wave in the solid. This quantity is sufficiently small so that $k_s d \ll 1$ and along with $k_x d \ll 1$, the solid displacement jump which gives $\boldsymbol{u}^s(0) \approx \boldsymbol{u}^s(d)$, may be cancelled. Yet another proposed approximation is to neglect the coupling between compression and tangential stresses cancelling T_{14} and T_{41} from the $\boldsymbol{T}(d)$ matrix. Finally, it is proposed to neglect the effect of the interstitial pressure on the in vacuo shear solid stress $\hat{\sigma}_{xz}$, canceling T_{15}. Note that these approximations lead to a much sparser $\boldsymbol{T}(d)$ matrix:

$$\boldsymbol{T}(d) \approx \boldsymbol{I} - d\boldsymbol{\alpha'} \approx \boldsymbol{I} - d\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\hat{A}^2-\hat{P}^2}{\hat{P}}k_x^2 - \tilde{\rho}\omega^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tilde{K}_{eq}} + \frac{k_x^2}{\tilde{\rho}_{eq}\omega^2} & 0 \\ 0 & -\tilde{\rho}_s\omega^2 & -\tilde{\rho}_{eq}\tilde{\gamma}\omega^2 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_{eq}\tilde{\gamma}\omega^2 & \tilde{\rho}_{eq}\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8}$$

As the simplifications do not reduce the number of parameters, it is in the following proposed to simplify the equivalent fluid quantities $\tilde{\rho}_{eq}$ and $\tilde{K}_{eq}$. Considering a thin layer, the tortuosity effects and set $\alpha_\infty \approx 1$, may be neglected. Using this last ansatz, it is proposed to use the following expressions:

$$\tilde{\rho}_{eq} \approx \frac{\rho_0}{\phi} + \frac{\sigma}{j\omega}, \qquad \tilde{K}_{eq} \approx \frac{\gamma_0 P_0}{\phi} \tag{9}$$

These last simplifications bring down the number of parameters in the model to six : three mechanical parameters ($\hat{A}$, N and the structural loss parameters η), the flow resistivity σ, the porosity ϕ and the bulk density ρ_1.

3 Validating the simplified model

A large range of configurations have been considered to validate the proposed approach. Particularly, a lot of attention has been paid to consider different backing conditions and incidence angles in order to assess the robustness of the simplified model. The three configurations shown on Figure 1 were used in the development process, and it was noticed that the backing featuring an air-gap (Figure 1.c) leads to results very similar to those obtained with a PEM backing (Figure 1.b) whereas the rigidly backed configuration (Figure 1.a) behaves differently. Only systems presented on Figure 1.a and 1.b are used to generate the figures displayed hereafter.

The numerical values for the films' parameters are taken from the literature [12]. For the figures presented in this paper a woven and a non woven film were used. The backing foam parameters' values are : $\phi = 0.994$, $\sigma = 9045\ \mathrm{N \cdot s \cdot m^{-4}}$, $\alpha_\infty = 1.02$, $\Lambda' = 197 \cdot 10^{-6}$ m, $\Lambda = 103 \cdot 10^{-6}$ m, $\rho_1 = 8.43\ \mathrm{kg \cdot m^{-3}}$ for the JCA parameters, $\nu = 0.42$ for the Poisson ratio, $E = 194 \cdot 10^3$ Pa for the Young's modulus and $\eta = 0.05$ for the structural loss factor. In the simulations, the

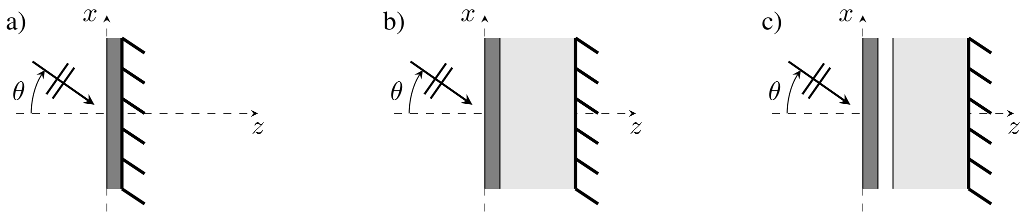

Figure 1: Configurations used during the method's development and for the tests.

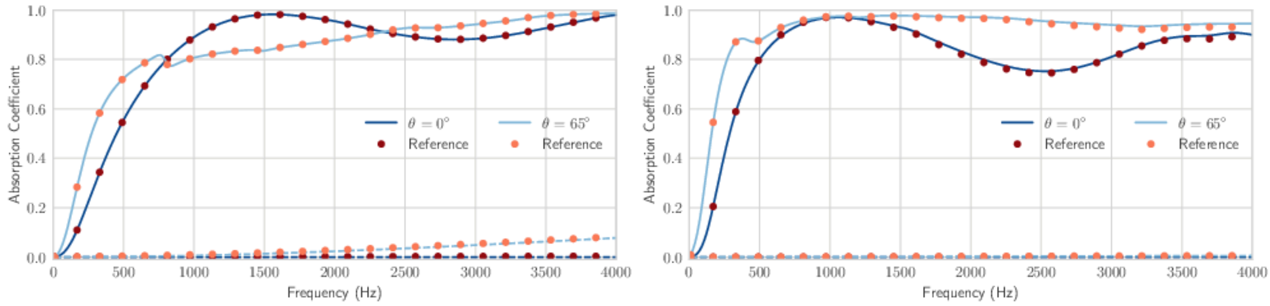

Figure 2: Absorption coefficient for two materials, two angles and two backing conditions as computed by the proposed approach (lines) and reference method (markers). The backing conditions are: PEM backing then rigid (solid lines) and rigid behind the film (dashed lines).

following properties were used for air: $\rho_0 = 1.213\ \mathrm{kg} \cdot \mathrm{m}^{-3}$, $\mathrm{Pr} = 0.71$, $\eta_0 = 1.839 \cdot 10^{-5}\ \mathrm{Pa} \cdot \mathrm{s}$, $P_0 = 1.01325 \cdot 10^5\ \mathrm{Pa}$, $\gamma_0 = 1.4$.

On Figure 2, the agreement between the proposed simplified model and a complete Biot-JCA model for the film is tested for two materials, two angles (0°and 65°) and two boundary conditions (rigid backing and PEM backing). As may be seen on the figure the results using the current modelling approach always stay close to the reference. The agreement is perfect even at low frequency and some minor discrepancies are observed higher in the spectrum.

Further testing of the sensibility of the model to different kinds of simplifications, lead to the following observations. The high frequency error on the absorption coefficient is linked to the simplifications on $\tilde{\rho}_{eq}$. Despite this, it is proposed to keep the model as is in order to remove $\tilde{\alpha}_\infty$ from the parameters.

In order to systematize the analysis, one may plot the evolution of the absolute error on the absorption using a complete Biot model as a reference in the (f, θ) plane. The error presented on Figure 3 is computed using:

$$\epsilon = |\alpha_{\mathrm{biot}} - \alpha_{\mathrm{screen}}| \tag{10}$$

where α_{biot} and α_{screen} are respectively the absorption coefficients calculated using a complete Biot-JCA model for the film and the simplified model.

Near grazing incidence, an expected loss of precision is observed, particularly when the film is directly laid on a rigid backing. This difference between the two setups is related to a a compensation effect in the backing PEM. Indeed, whereas the absorption occurs only in the film when directly laid on a rigid surface, it occurs mainly in the PEM in the other case, smoothing the effect of the film. At grazing incidence, the proposed model (where most of the shear effects have been neglected) struggles to represent how the physical phenomena leads to wave absorption. The path travelled by the wave in the media is also much longer, invalidating the hypothesis of a thin film. Despite these points the relative error is capped to less than 3% in the observed cases.

4 Conclusion

In the present work, a simplified model for the TM of acoustic films was proposed. It was shown that some approximations based on physical hypotheses lead to both a simpler propagation model (with expanded matrix exponential) and a reduction of the number of parameters. These simplifications hold for thin poroelastic layers and the resulting model

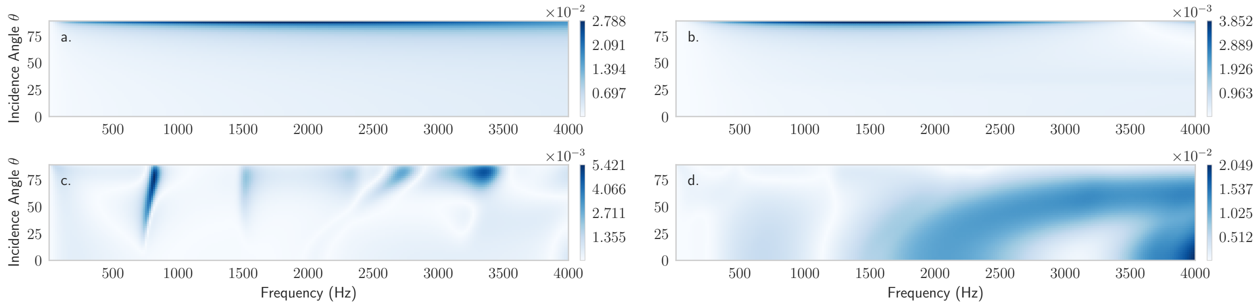

Figure 3: Evolution of the relative error in the (f, θ) plane. The first column (a. & c.) uses a non woven film, the second (b. & d.) a woven one. For the first line (a. & b.) the film is directly laid on a rigid backing, for the second (c. & d.) a PEM slab is inserted between the film and the backing.

behaves best when used in a transfer matrix model on top of another PEM or air layer. The model was tested as well on rigid backing and showed some discrepancies mainly due to some parameters being neglected, despite these aspects, the model still exhibits a reasonable precision also in these cases.

References

[1] C. Lagarrigue, J. P. Groby, V. Tournat, O. Dazel, and O. Umnova. Absorption of sound by porous layers with embedded periodic arrays of resonant inclusions. *The Journal of the Acoustical Society of America*, 134(6):4670–4680, 2013.

[2] N. Jiménez, V. Romero-Garcia, V. Pagneux, and J.-P. Groby. Quasiperfect absorption by subwavelength acoustic panels in transmission using accumulation of resonances due to slow sound. *Physical Review B*, 95(1), 2017.

[3] J. Yang, J. S. Lee, and Y. Y. Kim. Metaporous layer to overcome the thickness constraint for broadband sound absorption. *Journal of Applied Physics*, 117(17):174903, 2015.

[4] J. P. Parra Martinez, O. Dazel, P. Göransson, and J. Cuenca. Acoustic analysis of anisotropic poroelastic multilayered systems. *Journal of Applied Physics*, 119(8):084907, February 2016.

[5] L. Jaouen, F. Chevillotte, and F.-X. Bécot. Improving perforated plate modeling with porous media theory. *The Journal of the Acoustical Society of America*, 141(5):3797–3797, May 2017.

[6] M. A. Biot. Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid. *The Journal of the Acoustical Society of America*, 28(2):168, 1956.

[7] M. A. Biot and D. Willis. The Elastic Coefficients of the Theory of Consolidation. 1957.

[8] O. Dazel, B. Brouard, C. Depollier, and S. Griffiths. An alternative Biot's displacement formulation for porous materials. *The Journal of the Acoustical Society of America*, 121(6):3509, 2007.

[9] O. Dazel, J.-P. Groby, B. Brouard, and C. Potel. A stable method to model the acoustic response of multilayered structures. *Journal of Applied Physics*, 113(8):083506, 2013.

[10] Q. Serra, M. N. Ichchou, and J.-F. Deü. On the Use of Transfer Approaches to Predict the Vibroacoustic Response of Poroelastic Media. *Journal of Computational Acoustics*, 24(02):1550020, June 2016.

[11] J. P. Parra Martinez, O. Dazel, P. Göransson, and J. Cuenca. Derivation of the state matrix for dynamic analysis of linear homogeneous media. *The Journal of the Acoustical Society of America*, 140(2):EL218–EL220, August 2016.

[12] L. Jaouen and F.-X. Bécot. Acoustical characterization of perforated facings. *The Journal of the Acoustical Society of America*, 129(3):1400–1406, 2011.